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A COMPUTER PROGRAM FOR THE USE OF SENSITIVITY ANALYSIS IN DISPLAY EVALUATION

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ABSTRACT

The work reported herein was authorized under Contract NAS1-13734 in 1974. This study was conducted under the direction of Patrick Gainer, Simulation and Human Factors Branch, Langley Research Center, NASA.

This study was performed at Decision Science, Inc., San Diego, with Michael Mout and George Burgin acting as principal investigators.

This report provides a description of the Display Evaluation computer program, some results of this program and comparison of these results with a simple experiment. A detailed description of the experiment and data analysis is also included.

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SUMMARY

This report describes a technique of computer evaluation of a visual display or set of visual displays. The operator of a vehicle has many cues by which to estimate the state of his vehicle. This method deals only with a subset of the visual cues available. These cues come from various displays which reflect changes in the state of the vehicle.

These displays can reflect changes in any one individual state variable or combinations of changes in the state variables. The theory upon which this method is based is that the operator of the vehicle estimates the vehicle's state variable values by responding to changes in the display. It is assumed that the operator has perfect knowledge of the relationships between the display variables and the state variables. The only mistakes the operator makes are due to imperfection of his visual acuity in observing a given display. This acuity is assumed to be a function of where the observer is looking with respect to the center of the display. Mental errors such as control reversals cannot be measured by this method.

The result of this method is a covariance matrix and a correlation matrix. These matrices reflect the amount of expected error in estimating each of the state variables and the amount of ambiguity in the display between movement of two variables. These matrices allow the user to evaluate the various displays or sets of displays with respect to one another and also to evaluate various look points within a particular display. This evaluation may be used to indicate a general area that the pilot could scan in order

to limit the degree of error in estimation while minimizing the size of his scan path between several instruments.

The computer program for implementing this techniques has many options to allow the user as much freedom as possible in specifying the particular display or set of displays. The program at present is designed for four standard display types that might be seen in an aircraft. These are an eight-ball (for pitch, roll and yaw), an out-the-window display (such as a TV camera actually observing the real world), a circular dial type display, and a linear dial type display. There are many user specified options that may be used with these displays.

Preliminary theoretical results confirm intuitive feelings about displays. For example, a large display has less error than a small display, look points at the center of the display show less error and less ambiguity between variables than look points away from the center, also, variables displayed separately have less error and are not confused with other variables.

A simple experiment was conducted wherein a subject attempted to match up a movable cross with a fixed cross. This was done while fixating on various look points near the stationary cross. Results from this experiment were compared to a computer set-up of the same situation. This comparison tends to verify the technique for evaluating the worth of various look points within a display.

INTRODUCTION

The controller of a vehicle, particularly a remote controller, must derive information about the state of that vehicle from visual cues. These cues usually come from displays that respond in fixed ways to the changes in the state variables of the vehicle. These displays could respond to only one state variable, several variables or to all state variables. In any case the functional relationship between the display movement and changes in the state variables can be derived analytically.

The perfect controller of a vehicle with the perfect display or set of displays should be able to estimate the state variable values from the display variable values with no error. In other words the perfect controller sees exactly what the display variables are and knows exactly what state variable contributed to the display variables (this assumes unique display variable values for each possible state variable value). Mathematically this situation can be stated as follows: The state variable/display variable function is one to one, the controller has absolute knowledge of this function, and the controller perceives the display without error.

When a nonperfect controller with a nonperfect display makes an error in estimating the state variable values it can be due to any of three causes. One of these three causes, lack of absolute controller knowledge of the display state variable function cannot be directly measured and can change from controller to controller depending upon learning capability, experience, coordination, and so forth. The other two causes that could contribute to errors are a nonperfect display (display/state variable function is not one to one) and the lack of the controller to perceive the display variable values exactly. Both of these causes can be measured. The display/state variable function is either known exactly or can be approximated to some degree of accuracy. The ability of a controller with normal vision to perceive movement or distance is

called visual acuity and has been measured to a certain degree of accuracy for various visual angles. Using this acuity information and the relationship between the display and state variables a measure can be derived of the accuracy that could be expected from a perfectly trained controller. This measure would be dependent on some specific look point for each display.

In 1973 a working paper (LWP1131) was prepared by Patrick A Gainer which laid the theoretical basis for this technique. This report describes a computer program which implements this technique for a wide variety of applications. This program is general enough to accept many kinds of input displays. It enables the user to evaluate various displays as he desires. The output from this program gives the user a measure of the relative value of the input display and of various look points within that display. Optionally this program includes maximum likelihood estimates of the state vector.

Other tasks accomplished in connection with this computer program were to obtain experimental comparison data. These data are used to verify the worth of this technique in applying it to real world problems.

LIST OF SYMBOLS

α_{ij}	Horizontal angle (longitude) for location of the center of display segment i,j; $i = 1,2,\ldots n$ $j = 1,2,\ldots m$
β_{ij}	Vertical angle (latitude) for location of the center of display segment i,j
$\underline{X}$	Vector of state variables
nv	Number of state variables
$\underline{X}'$	(' denotes transpose) = $(x_1, x_2, \ldots x_{nv})$
$\Delta\alpha_{ij}$	Change in horizontal angle of segment i,j due to change in $\underline{X}$
$\Delta\beta_{ij}$	Change in vertical angle of segment i,j due to change in $\underline{X}$
$f(\underline{X};\alpha,\beta)$	Function relating changes in $\underline{X}$ to a new horizontal angle for the segment centered at α,β
$g(\underline{X};\alpha,\beta)$	Function relating changes in X to a new vertical angle for the segment centered at α,β
$f_{ij}(\underline{X})$ =	$f(X;\alpha_{ij},\beta_{ij})$
$g_{ij}(\underline{X})$ =	$g(X;\alpha_{ij},\beta_{ij})$
I	Influence matrix of partial derivatives of f_{ij} and g_{ij} with respect to all state variables
$\underline{0}$	Vector of the actual values of α_{ij} and β_{ij}

$\Delta\underline{0}$	Vector of the actual values of $\Delta\alpha_{ij}$ and $\Delta\beta_{ij}$ for a given change in $\underline{X}$
$\underline{0}'$	$(\alpha_{11}, \beta_{11}, \alpha_{12}, \ldots, \alpha_{nm}, \beta_{nm})$
$\Delta\underline{0}'$	$(\Delta\alpha_{11}, \Delta\beta_{11}, \Delta\alpha_{12}, \ldots, \Delta\alpha_{nm}, \Delta\beta_{nm})$
$\hat{\underline{0}}$	An estimate of $\underline{0}$
$\mathrm{cov}(\hat{\underline{0}})$	Covariance matrix of the estimate of $\underline{0}$
$\underline{X}$	Estimate of $\underline{X}$ derived from $\hat{\underline{0}}$
λ_{ij}	Acuity for segment i,j due to a given look point (See Graph 1 for the assumed acuity function used in this program.)

DESCRIPTION OF THE METHOD

Basically this method assumes that the observer/controller's visual world can be thought of as a sphere of unit radius. All of the visual cues that are available to the observer can be thought of as projected onto this sphere. This sphere (or a section of this sphere which contains information) can be divided into small rectangular segments (for example, 1° by 1°). Each of these segments can possibly move whenever changes in the state variables occur. If very small segments are used then only vertical and horizontal movement need be considered. For small changes in the state variables the functional relationships between the segment movement and the state variables can be linearized. Thus the display/state variable function will be linear. The display variables are the horizontal and vertical changes of each segment. The linearized display/state variable function can be represented by the matrix I. This matrix contains the partial derivatives of the display variables with respect to the state variables. These derivatives are of the actual display/state variable function (analytically derived) at the values of the state variables being considered. So in effect each element of the matrix I relates the relative change in a specific state variable to the horizontal or vertical displacement of a specific display segment. If we let

$$f_{ij} = f(\underline{X};\alpha_{ij},\beta_{ij})$$

and

$$g_{ij} = g(\underline{X};\alpha_{ij},\beta_{ij})$$

where g_{ij} and f_{ij} are functions relating changes in $\underline{X}$, the state vector, to change in the angles α_{ij}; and β_{ij}.

Then the influence matrix I is given by:

$$
I = \begin{bmatrix}
\frac{\partial f_{11}}{\partial x_1} & \frac{\partial f_{11}}{\partial x_2} & \cdots & \frac{\partial f_{11}}{\partial x_{nv}} \\
\frac{\partial g_{11}}{\partial x_1} & & & \frac{\partial g_{11}}{\partial x_{nv}} \\
\frac{\partial f_{12}}{\partial x_1} & & & \\
\vdots & & & \\
\frac{\partial f_{nm}}{\partial x_1} & & & \\
\frac{\partial g_{nm}}{\partial x_1} & & & \frac{\partial g_{nm}}{\partial x_{nv}}
\end{bmatrix}
$$

If the state vector $\underline{X}$ is multiplied by the matrix I, the result is a vector $\underline{0}$ of observations of horizontal and vertical movement of each segment.

$$\underline{0} = I\underline{X}.$$

Now if $\underline{X}$ changes by value, say ΔX, the corresponding observation change $\Delta\underline{0}$ would be

$$
\Delta\underline{0} = \begin{pmatrix}
\Delta\alpha_{11} \\
\Delta\beta_{11} \\
\Delta\alpha_{12} \\
\Delta\beta_{12} \\
\vdots \\
\Delta\alpha_{mn} \\
\Delta\beta_{mn}
\end{pmatrix}
$$

where

$$\Delta\alpha_{11} = \frac{\partial f_{11}}{\partial x_1} \Delta x_1 + \frac{\partial f_{11}}{\partial x_2} \Delta x_2 + \ldots + \frac{\partial f_{11}}{\partial x_{nv}} \Delta x_{nv}$$

$$\Delta\beta_{11} = \frac{\partial g_{11}}{\partial x_1} \Delta x_1 + \frac{\partial g_{11}}{\partial x_2} \Delta x_2 + \ldots + \frac{\partial g_{11}}{\partial x_{nv}} \Delta x_{nv}$$

$$\vdots$$

$$\Delta\alpha_{nm} = \frac{\partial f_{nm}}{\partial x_1} \Delta x_1 + \ldots + \frac{\partial f_{nm}}{\partial x_{nv}} \Delta x_{nv}$$

$$\Delta\beta_{nm} = \frac{\partial g_{nm}}{\partial x_1} \Delta x_1 + \ldots + \frac{\partial f_{nm}}{\partial x_{nv}} \Delta x_{nv}$$

Now this change in $\underline{0}$ is observed by the viewer of this display. This user wants to infer how this change in $\underline{0}$ was caused by a change in $\underline{X}$. To do this the viewer estimates the new value of $\underline{0}$; call this estimation $\hat{\underline{0}}$. Let

$$\hat{\underline{0}} = I\underline{X} + \underline{\xi}$$

where $\underline{\xi}$ is the error of estimation. The eventual goal is to derive an estimate of $\underline{X}$ from this estimate of $\underline{0}$. Also, the distribution of the $\underline{X}$ estimate is needed in order to establish error limits on this estimation. To do this a multinormal distribution is assumed for $\underline{\xi}$, a transformation of $\underline{\xi}$ is required to get a best estimate of $\underline{X}$ (in a least squares sense) that has known distribution parameters.

From the above equation for $\underline{0}$

$$\underline{\xi} = \hat{\underline{0}} - I\underline{X}$$

Let the multinormal Gaussian distribution of $\underline{\xi}$ have zero mean vector and a diagonal covariance matrix $\mathrm{cov}(\hat{\underline{0}})$. The diagonal

elements are the variances of the error in estimating the horizontal and vertical position of the center of each segment. The standard deviation (square root of the variance) of this error is determined by the reciprocal of the observer's acuity in that part of his visual field. Both horizontal and vertical acuity are considered to be the same for a given segment, so that if λ_{ij} is the acuity for segment i,j then

$$\mathrm{cov}(\hat{\underline{O}}) = \begin{pmatrix} \lambda_{11}^{-2} & 0 & 0 & 0 & & & & 0 \\ 0 & \lambda_{11}^{-2} & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \lambda_{12}^{-2} & 0 & & & & \vdots \\ 0 & 0 & 0 & \lambda_{12}^{-2} & & & & \vdots \\ & \vdots & & & \ddots & & & \vdots \\ & \vdots & & & & & \lambda_{mn}^{-2} & 0 \\ 0 & 0 & & & & & 0 & \lambda_{mn}^{-2} \end{pmatrix}$$

Now let $\mathrm{cov}(\underline{O})^{-1/2}$ be the matrix such that

$$\mathrm{cov}(\hat{\underline{O}})^{-1/2} \ \mathrm{cov}(\hat{\underline{O}})^{-1/2} = \mathrm{cov}(\hat{\underline{O}})^{-1}.$$

$\mathrm{Cov}(\hat{\underline{O}})^{-1/2}$ is the matrix with the acuity values λ_{ij} on the diagonal.

If equation $\underline{\xi}$ is multiplied by $\mathrm{cov}(\hat{\underline{O}})^{-1/2}$, the resultant re-sidual vector $\mathrm{cov}(\hat{\underline{O}})^{-1/2}$ $(\hat{\underline{O}}-I\underline{X})$ weights each segment displacement by the acuity of that segment. The residuals $\hat{\underline{\xi}}$ of this equation are now of the form

$$\hat{\underline{\xi}} = (\mathrm{cov}(\hat{\underline{O}})^{-1/2})\underline{\xi} = (\mathrm{cov}(\hat{O})^{-1/2})(\underline{O}) - (\mathrm{cov}(\hat{O})^{-1/2}) \cdot (I\underline{X}).$$

and $\hat{\underline{\xi}}$ has a multinormal distribution with zero mean and a covariance matrix equal to the identity matrix. This equation for $\hat{\underline{\xi}}$ can now be used to find a best estimate of $\underline{X}$. Call this estimate $\hat{\underline{X}}$, and let it minimize the sums of squares of these weighted residuals, $\hat{\underline{\xi}}$ or

$$\min_{\hat{\underline{X}}}(\hat{\underline{\xi}}'\hat{\underline{\xi}}) = \min_{\hat{\underline{X}}}\{(\mathrm{cov}(\hat{\underline{O}})^{-1/2}\hat{\underline{O}} - \mathrm{cov}(\hat{\underline{O}})^{-1/2}I\hat{\underline{X}})'(\mathrm{cov}(\hat{\underline{O}})^{-1/2}\hat{\underline{O}} - \mathrm{cov}(\hat{\underline{O}})^{-1/2}I\underline{X})\} = \min_{\hat{\underline{X}}}\{(\hat{\underline{O}} - I\hat{\underline{X}})'\ \mathrm{cov}(\hat{\underline{O}})^{-1}(\hat{\underline{O}}-I\hat{\underline{X}})\}.$$

Solving the resulting normal equations gives

$$\hat{\underline{X}} = [I'\mathrm{cov}(\hat{\underline{O}})^{-1}\ I]^{-1}\ I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}\ \hat{\underline{O}}.$$

Now if $\hat{\underline{X}}$ is of the form $\hat{\underline{X}} = A\hat{\underline{O}}$ then

$$\mathrm{cov}(\hat{\underline{X}}) = A\ \mathrm{cov}(\hat{\underline{O}})\ A'$$ (ref. 1, p. 79).

Here

$$A = [I'\mathrm{cov}(\hat{\underline{O}})^{-1}\ I]^{-1}\ I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}$$

then

$$\mathrm{cov}(\hat{\underline{X}}) = [I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}\ I]^{-1}\ I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}\ \mathrm{cov}(\hat{\underline{O}})$$
$$(I'\ \mathrm{cov}(\hat{\underline{O}})^{-1})^{-1}\ [I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}\ I]^{-1}$$

and since $\mathrm{cov}(\hat{\underline{O}})$ is a symmetric matrix this reduces to

$$\mathrm{cov}(\hat{\underline{X}}) = [I'\ \mathrm{cov}(\hat{\underline{O}})^{-1}\ I]^{-1}.$$

This is basically the method that the computer program is written to calculate.

THE COMPUTER PROGRAM

This program has evolved through several stages but will be described here in its final form. The program is basically written for a vehicle that is moving in space with possible movement in six degrees of motion (three Euler angles and the three translation axes). This movement results in changes of segments on a sphere of unit radius. These changes are dependent on specified functional relationships between the sphere segments and vehicle movement.

In order to better understand the workings of this program a short example will be considered. Let the controller be seated one meter from his display panel (see Figure 1). At this distance 1.75 cm. subtends approximately 1 degree of arc. On this panel are two displays. One display shows range x on a linear scale. This display is approximately 5 cm. high and 23 cm. wide, with a moving pointer to show relative range. Another display which is 60 cm. wide by 40 cm. high is a TV camera showing the outside view of the vehicle. The center of the panel is located straight ahead of the controller. The center of the linear display is located 33 cm. to the left of the center and 16.5 cm. down. The center of the TV display is located 24 cm. to the left and 33 cm. above this center line.

The computer program will generate measures of how well the controller can estimate change in these displays from some nominal state variable conditions. These measures are the amount of expected error and the confusion measure (correlation) between state variables.

This example is shown in more detail further on in this chapter along with the corresponding computer set-up and output.

The first task performed by the computer program is to set up the display or displays (two or more displays may be considered simultaneously). This set-up is for the display size, location,

and segmentation. Each display has a specified function that relates segment movement (vertical and horizontal) in that display to the state vector changes. This movement may be in response to only one state variable, as in an altimeter, or to all state variables, as in an out-the-window view or a TV camera view.

When this set-up has been completed, the program then calculates the elements of the influence matrix. These elements relate the changes of each state variable to changes in the vertical and horizontal position of each segment. The partial derivatives here are found by numerical differentiation.

Now the program is ready to read in a series of look points for each display in this set-up. These viewpoints can be anywhere on the sphere but are usually located near the display center being referenced. Once the look points for each display is read in the visual acuity values can then be calculated for each segment. The values are a measure of how well the observer can estimate either displacement (position) or rate-of-movement of that particular segment while fixating on the given look point. These acuity values are used to calculate the covariance matrix of the observation vector which, in turn, is used in calculation of the covariance matrix of the state vector. The covariance matrix of the state vector along with the correlation matrix are then output for the user's reference.

The program optionally generates a series of perturbations in the state vector and calculates the maximum likelihood estimate (MLE) of each of these perturbations. This procedure can be repeated for the various look points in this display set-up.

Various options are available allowing the user to specify the display size, segment size, functional relationships, size of state vector perturbations, step-size in the numerical differentiation, overall acuity weights for a particular display (this to reflect possible degradation of a display either visually or mechanically) weights for individual look points in a display set (to

reflect memory retention or lack of retention), and individual segment weights within a display (to reflect the amount of useful information or cues in that segment). Furthermore, this program is set up to allow the user to program his own special display/state variable functions with a minimum of changes in the total program. New acuity functions may also be introduced.

An Example

Tables 1 through 5 show the set-up and results of the example discussed at the beginning of this section.

The first display is a linear display of variable 4 (range or "X"). This display moves horizontally as X changes. This display is 14° by 3° with segments size 2° by 1°. The first display is located to the left and down from the observer's center. The second display is a real world picture. It is located to the left and up. This display is 35° by 25° in size with segment size of 5° by 5°. This display shows information from a camera or periscope pointing 20° to the right of the center of the vehicle. The picture being portrayed is degraded (this to model a TV with too few scan lines, a dirty camera lens, clouds or haze, etc.), and has an acuity weight of .6827. Tables 3 through 5 show the results from the first set of look angles for this display set. Table 3 shows the look angle and the final weight for each display. The final weight for look system (display) two now is .6144. This is less than the original acuity weight of .6827. The reason for this is that the final weight is the product of the acuity and the memory retention weight. In this case display two has a .9 memory weight given a final display weight of (.6827)(.9) = .6144.

This display set has the option to read-in the segment weights. In display 1 the weighted segments are the second column of segments. This is to indicate a pointer on the linear scale. Display 2 has weighted segments in the middle of the display. This indicates that the center of this display contains more information

than the peripheral areas. The resulting correlation matrix from this analysis shows that variable 4 ("X" or range) has a much lower standard deviation and has no strong correlation with any of the other variables. This is because the "X" variable is the only variable on the linear display (display 1)

In these tables of results there are eigenvector analysis results. These matrices can be used to find an orthogonal space to generate new variables that are independent. These matrices have no immediate use in the present analysis but their meaning is being investigated for future use.

EXPERIMENTAL VERIFICATION

A search was made for experimental data that could be used to verify this particular display evaluation method. In the past, displays have been evaluated by comparing performance between two or more displays. This performance has usually been measured on some analog type device giving some total measure at the end of a run. The data needed for experimental comparison to this method had to be in some form of multivariate observations from which a covariance matrix could be calculated. Also, information on the look angle for these observations had to be available. No experimental data could be located which satisfied these criteria. Due to this, a simple experiment was designed and conducted at the Decision Science, Inc. facility. A full description of this experiment and the results follows. A summary of these results will be repeated here.

Summary of Results

The theoretical covariance matrices were calculated for two different weighting schemes on four experimental conditions. The first weighting scheme was a gross approximation of the actual display and resulted in theoretical variances which were much lower than the sample variances. The second weighting scheme was an attempt to more precisely reflect the actual display. This second weighting scheme resulted in much larger theoretical variance values. The values were in many cases <u>not</u> significantly different from the sample variances. Under both weighting schemes most of the sample correlations were not significantly different from the theoretical.

Experimental Conclusions

The primary conclusion to be drawn from these results is that the basic computer model of the experiment can drastically affect the results. When using the proper setup, the algorithm seems to be a valid model of human performance. There are many consistencies noted between the sample covariance matrices and the theoretical matrices. The presence of the consistencies give strong indication that further experimental data would be valuable for a more in-depth verification of this method.

The Experiment

A simple experiment was conducted as follows:

Display.- An overhead projector showed a cross to simulate the pitch, roll and yaw (θ, ϕ, and ψ) of some vehicle (see Figures 2A, 2B, 2C).

Look Point.- The observer was instructed to look at some point (a dot on the display, see Figures 2A, 2B, and 2C) in this display.

Estimation of State Variables.- The subject then placed a moveable crosshairs in line with the displayed crosshairs while continuously looking at the dot. If the subjects look point changed while the subject attempted to match up the crosshairs, the data were discarded.

Measurement.- After the observer estimated the state variable, measurements of error of these estimates were taken (see Figures 2B and 2C). These values were converted to degrees and minutes of arc. The errors were then used to get an estimate of the covariance matrix which was compared to the original matrix produced by the method being tested.

Experimental Results and Comparison to Theoretical Data

Subjects and Data Collection.- One subject was used on this experiment with two different display configurations and two look points for each display. There were 12 observations taken under each of these four conditions. A different subject was tested under the first condition (11 observations) in order to verify that there did not exist any great subject variations.

In order to derive the theoretical results each experimental condition was converted to a set-up compatible with the computer program. This basically involved transforming the data on look points, display size, and so forth into degrees of arc. Also, the actual display had to be put in a segmented form.

The four experimental conditions and the respective computer set-ups are shown in Figure 3A, 3B, 3C, and 3D. The distance from the subject to the displayed view was 165.0 cm. This distance resulted in a display of 17° x 17° with segments of 1° x 1°. The various weights for the segments and look point locations are shown in Figures 3B and 3D. The output from the computer program shown in Tables 6 through 14 gives the set-up in numerical form and also the resultant theoretical distribution parameters.

Data Preparation.- A computer program was written to convert the observation data from distance to minutes of arc then calculate the corresponding covariance matrices for each of the experimental conditions. The conversion from distance measures to minutes of arc was accomplished by the formula:

$$\text{min of arc} = 60 \times \text{TAN}^{-1}(\ell/d)$$

where ℓ = length to be converted

and d = distance from the viewer to the picture

The distance to the picture in this case was 165 centimeters (cm.) For example, in Figure 2A the cross length is 42 cm. which converts

to

$$857 \text{ min of arc} = 60 \times \mathrm{TAN}^{-1}(42/165)$$

or approximately 14° as shown in Figure 2B.

The program also calculates the rotation angle, ϕ, from the change in slope of the estimated crosshairs with respect to the displayed crosshairs; this is done over a 10 cm. length. Here two measures 10 cm. apart, A and B (see figure 2C), of the vertical distance from the display horizontal axis to the estimated horizontal axis are taken. The angle ϕ is then found by the following formula

$$\phi = 60. \times \mathrm{TAN}^{-1}((A-B)/10.).$$

The results of this program are shown in Tables 15 through 19. Tables 15 and 16 are for the same experimental conditions with two different subjects. Tables 17, 18 and 19 show results from the first subject under three different experimental conditions (see Figures 3A, 3B, 3C, and 3D for experimental conditions). These tables also show the experimental data used to derive each of the matrices.

Analysis and Discussion

The results comparing the two different subjects using display A, look point 1, show no significant difference in the covariance matrices. Using Box's test for equality of covariance matrices (Ref. 1, p. 158), a chi-squared value of 7.82 (six degrees of freedom) is calculated. This value has a corresponding p-value of .25 (greater than .05) under the hypothesis of equal covariance matrices. The mean vector values are easily within the one mean standard deviation of zero so that the conclusion of no significant subject difference is made. Due to this result the remaining tests were run with only one subject, this being due to time, personnel, and cost restraints. (Tables 22 and 23 show the data from these two subjects.)

In comparing the theoretical results with the experimental results several important items should be noted. Of prime importance are the restrictions that the computer program places on the model. The true condition cannot be reflected exactly, particularly in the segmentation and weights. For simplicity's sake, a segment was given a weight of one if a line goes through the middle of it and a weight of 1/2 if the line crosses a corner (see Figures 3B and 3D). It is possible that smaller segment sizes (requiring programming changes) or more carefully calculated weights would give results closer to the real world experiment.

Table 6 shows the display set-up. Tables 7 through 14 have the preliminary theoretical results. Tables 15 through 19 show the experimental data and results.

In general, the experimental results show much higher variances as opposed to the theoretical results (see Table 28). This seemed to be due in a large part to the grossness of the segment weights for the different displays. Because of this the segment weights were calculated to more realistically reflect the actual display. This was done by carefully drawing the real display in the segmented sections. The weights were then calculated for each segment as the percentage of that segment that was filled. This resulted in the segment weights shown in Figures 4A and 4B. Here the width of the display lines was taken to be 3.2 mm. or .11 degree of arc. Since the segments were $1^{\circ} \times 1^{\circ}$ a segment with two lines through it (see the center segment of Figure 4B), each 1° long, would have a .22 weight.

The results of this weighting scheme are shown in Tables 20 through 27. Here the theoretical variances are much larger than with the previous weights and in many cases are not significantly different from the experimental results (see Table 28, graph 2, and graph 3 for these comparisons). This new weighting scheme changes the correlations somewhat, but not drastically. Several of the sample correlations were significantly different from the theoretical, but in most cases the differences were small (see

Table 29 for comparison and statistics on correlation).

The comparisons between look points in a given display and within the covariance matrices for a given look point are for both the theoretical and experimental results. The experimental data shows larger variances on the ϕ variable than θ and ψ. In general, the variance of the experimental data decreases as the look angle decreases. Both of these results are seen in the theoretical parameters.

Overall, the results of this simple experiment seem to justify the use of this algorithm for evaluating the comparative worth of different displays and different look points within a display. Further experimentation with more observations per look point, more variety of look points, displays of varying acuity and use of more subjects would be justified for a more absolute verification of the evaluation techniques.

CONCLUSIONS

The basic conclusion to be made from this study is that the technique described herein is a valid technique for evaluating displays. The computer program generates results on simple displays that are agreeable with intuition. The computer program is presently flexible enough to allow the user to investigate a wide variety of combinations of displays. These displays may be analyzed as being viewed simultaneously or sequentially. The program can be modified to include special types of displays not presently programmed.

Overall, the computer program of this technique can be utilized to evaluate the relative worth of any number of theoretical displays without actually constructing these displays. Using this program for calculating the absolute worth of a display would require more careful verification of the technique and certainly a precise modeling method of the actual display.

The basic problem in applying this technique to human performance is the nonaccountability of mental errors. This would require the actual modeling of the mental reaction pattern of the subject in question. The best that can be expected of this technique is to give the best limitations of the performance that could be expected from the perfect human operator.

RECOMMENDATIONS FOR FUTURE WORK

Primarily, the item most lacking in reference to this technique is in-depth experimental verification. This would require an experiment explicitly designed to generate data compatible with this technique. The ideal situation for this type of experiment would require an eye-tracking device and a computer interface with some sort of display/control equipment.

Further extensions of the computer program could involve the inclusion of more display/state variable functions, these to possibly reflect relationships other than vehicle movement. Other possible additions could be that of deriving and generating further evaluation measures based upon the eigenvector analysis now included in the program.

The most promising extension of this technique is to expand the program to include displays that are changing with time. This would include the expansion of the technique to update the covariance matrix and state vector estimation with each change in look point and display change. The results of this extension to the algorithm could be used to find the limitations of a pilot in a nonstatic situation. These limitations would be expressed in the amount of error that could be expected due to certain aircraft maneuvers and scanning patterns of the displays.

Decision Science, Inc.
San Diego, California, July 6, 1976

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3. Rice, R. F.: "An Advanced Imaging Communication System for Planetary Exploration 'PostPrint,'" SPIE Seminar Proceedings, Vol. 66, August 1975.

4. Erickson, Robert A.: Human Factor Research Techniques with Television, reprinted in Report NWC TP 5072, Weapons Development Department, Naval Weapons Center, China Lake, California, January 1971.

5. Ralston, Anthony: A First Course in Numerical Analysis, 1965, P. 47.

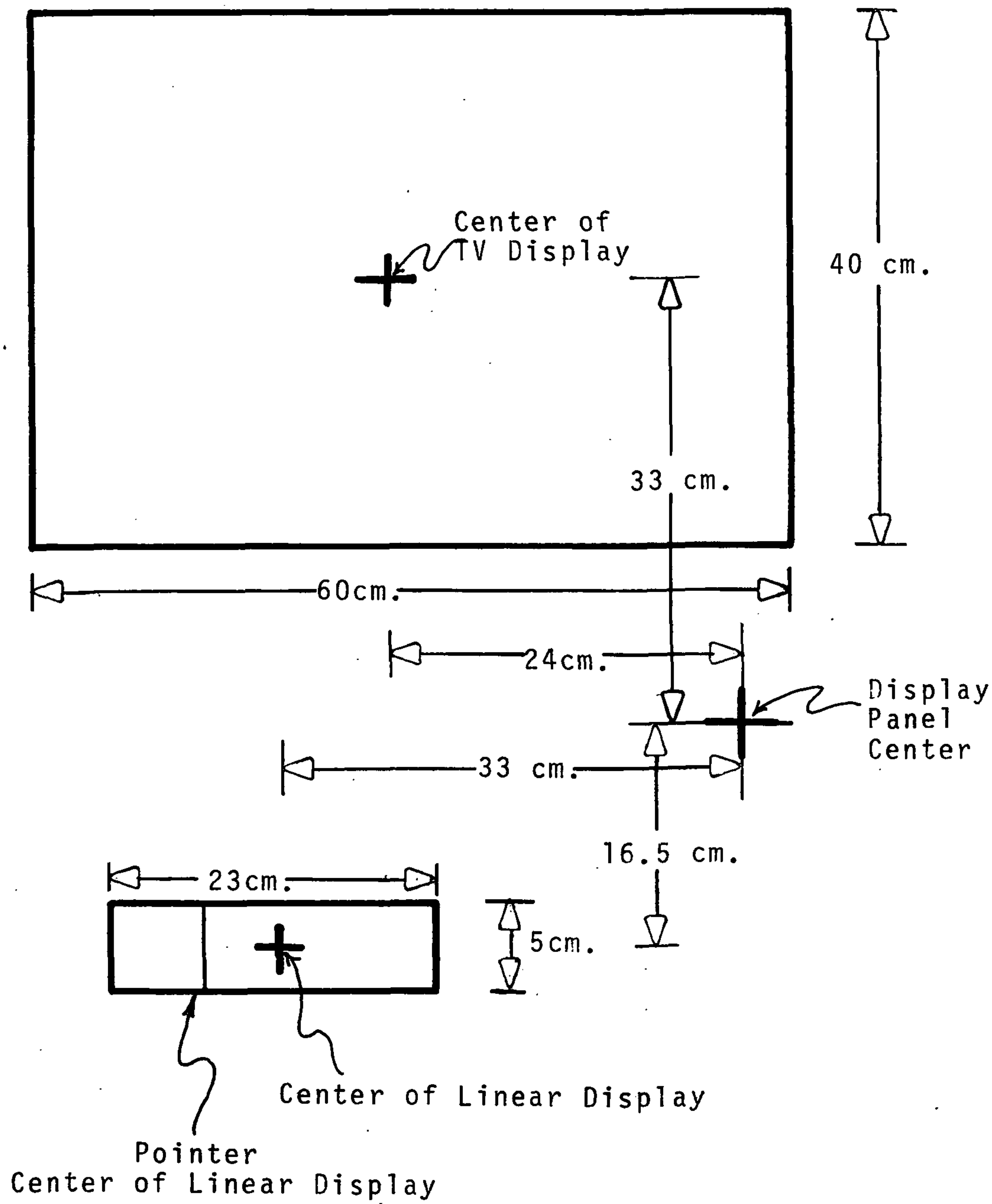


Figure 1.- Example of a Display Panel with Two Displays

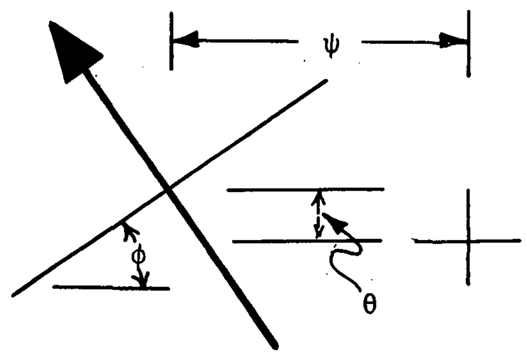


Figure 2A.- Display as seen by Observer

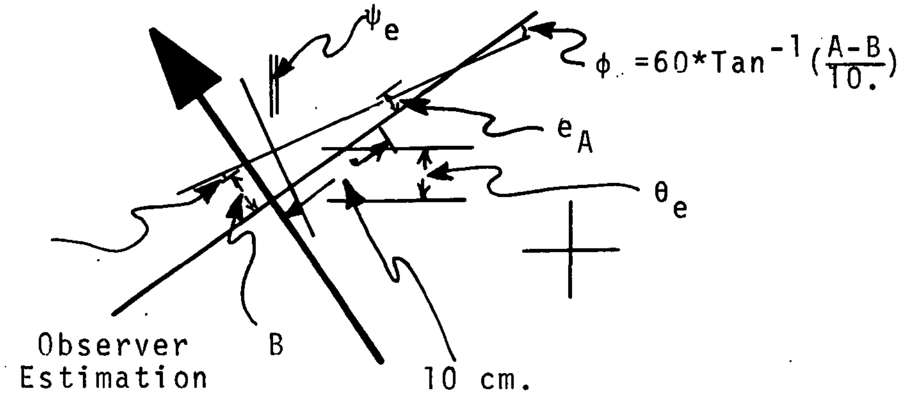
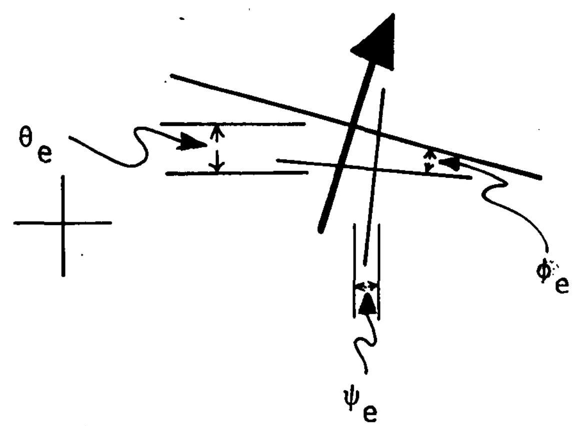


Figure 2B.- Display with Observer Estimate

Figure 2C.- Display with Observer Estimate and Error



Figures 2A, 2B and 2C.- Displays and Observer Estimates

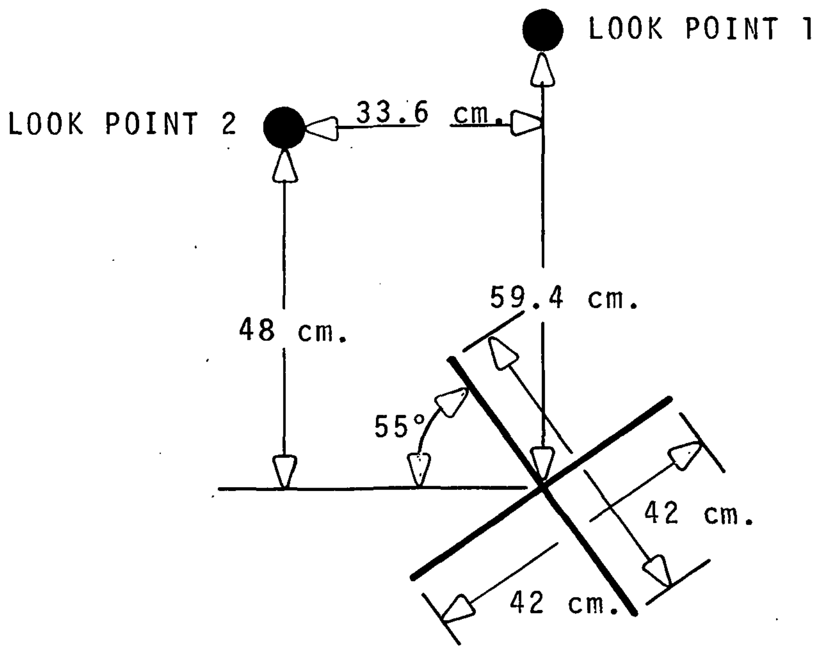


Figure 3A.- Configuration of Display A

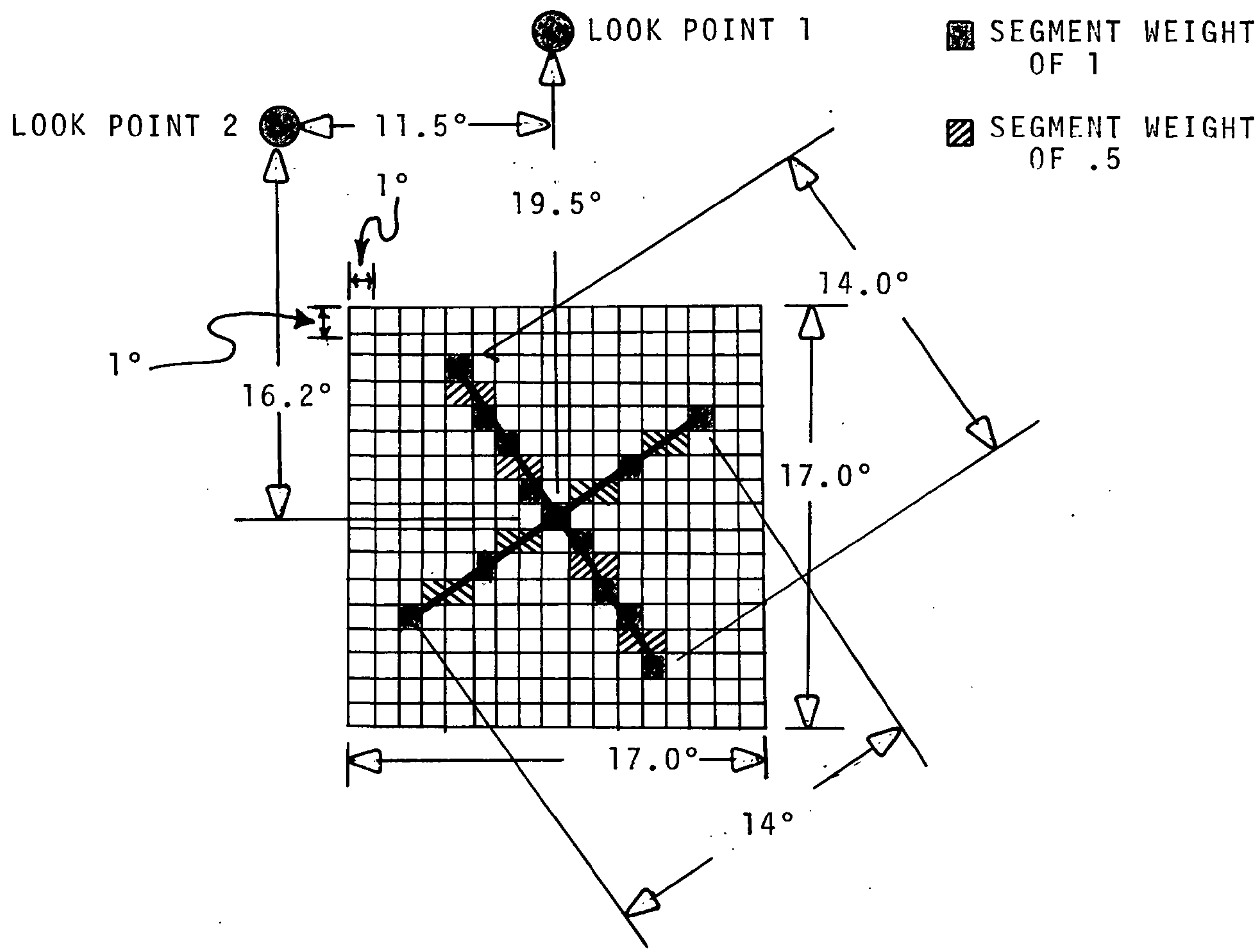


Figure 3B.- Computer Set-up of Display A with Lookpoints 1 and 2

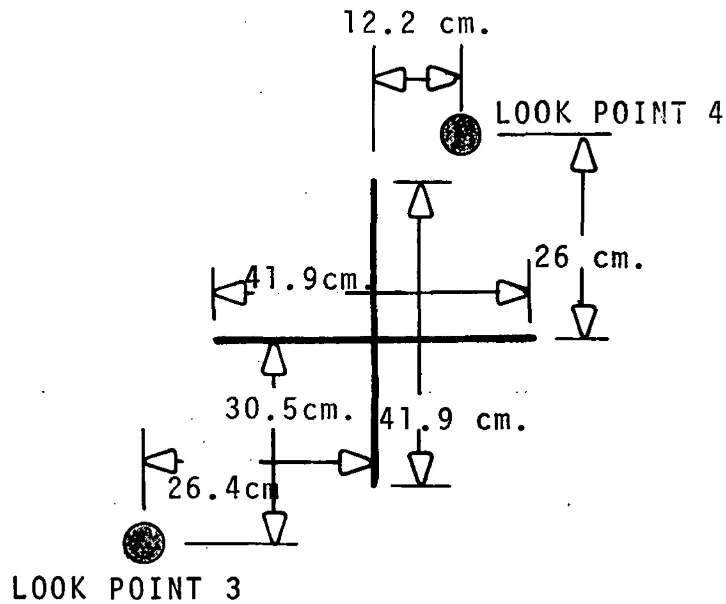


Figure 3C.- Configuration of Display B

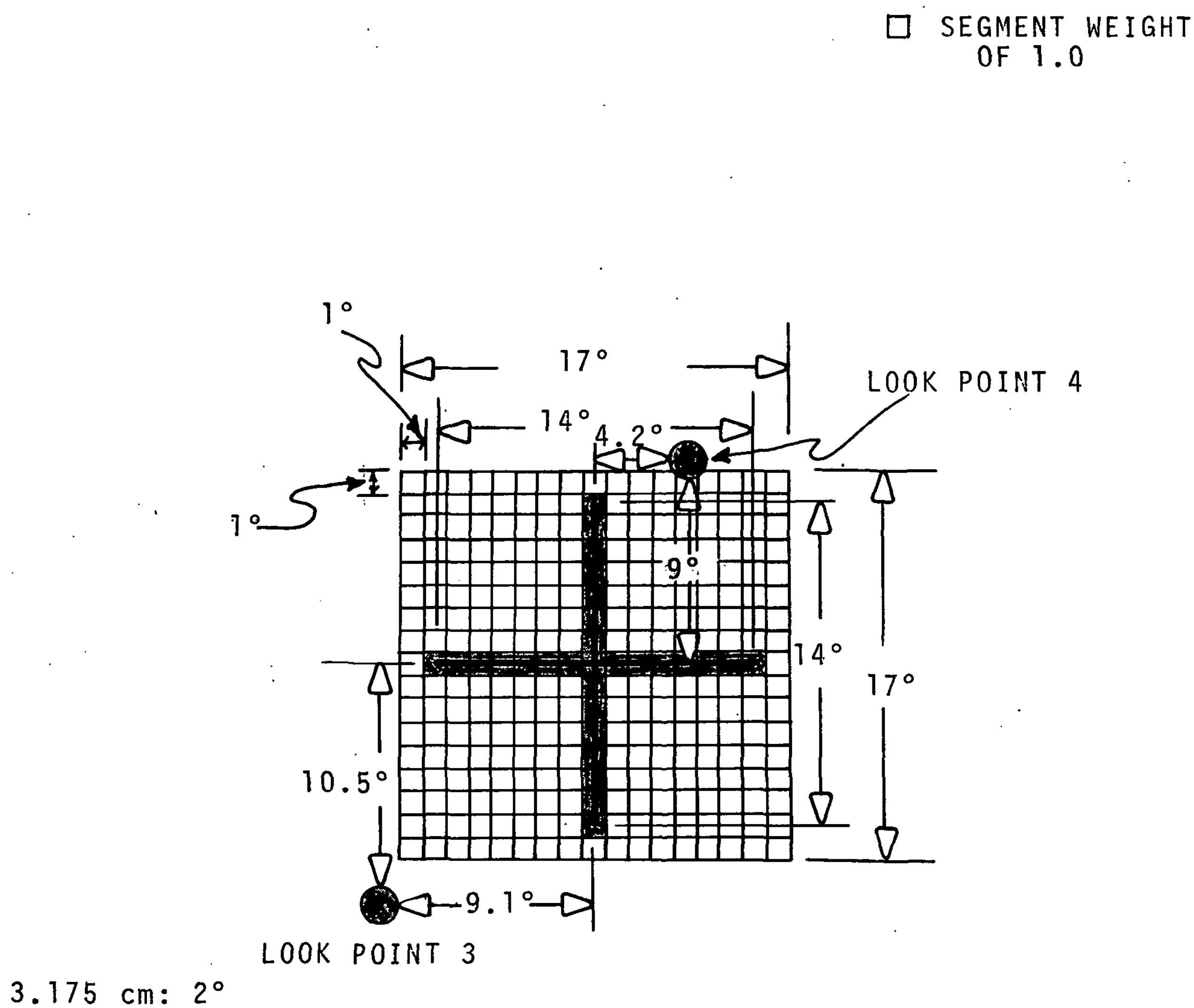


Figure 3D.- Computer Set-up for Display B with Look Points 3 and 4

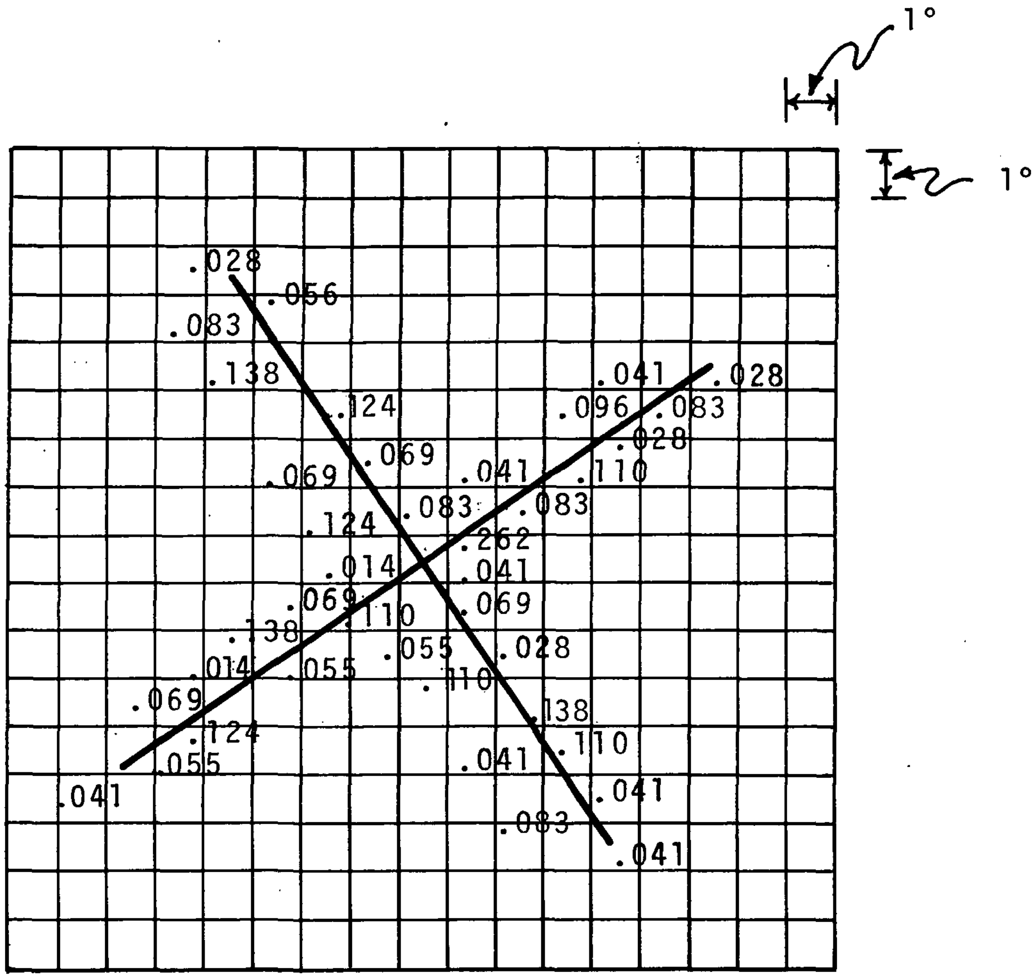


Figure 4A.- Fine Weights - Display A

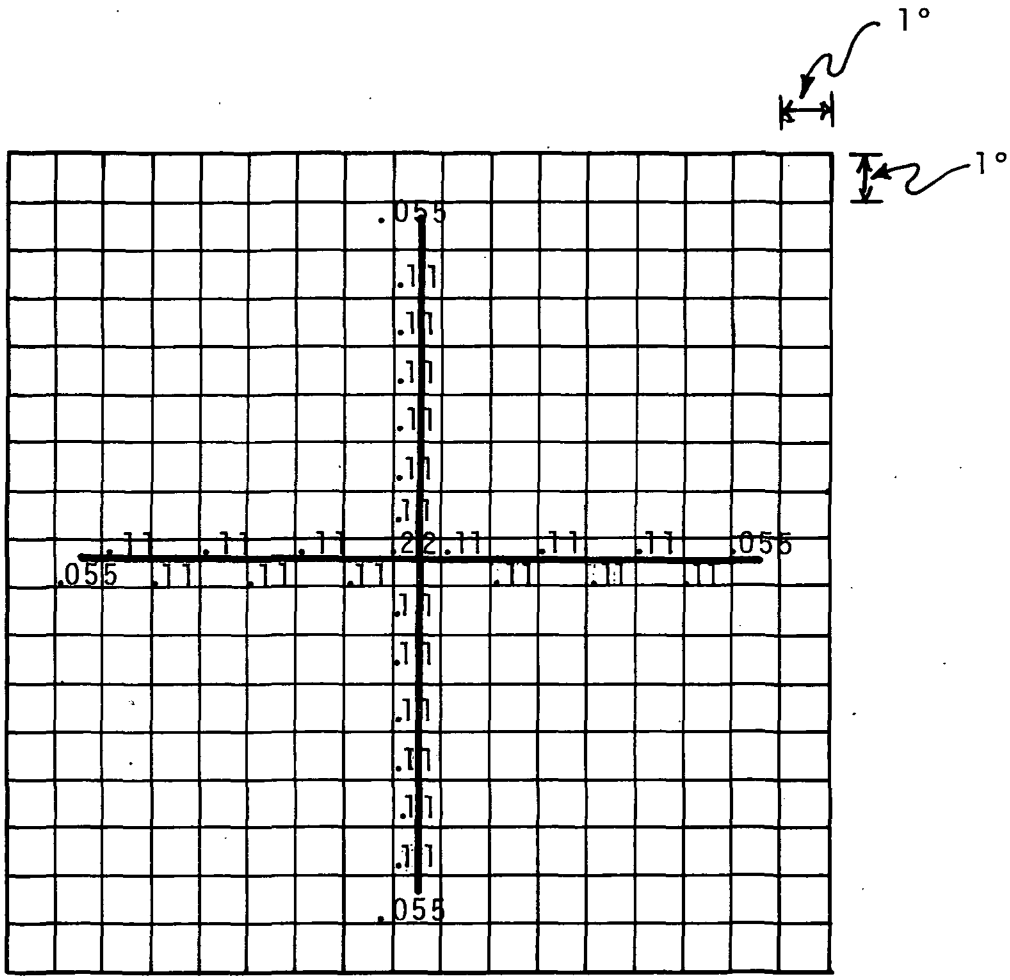
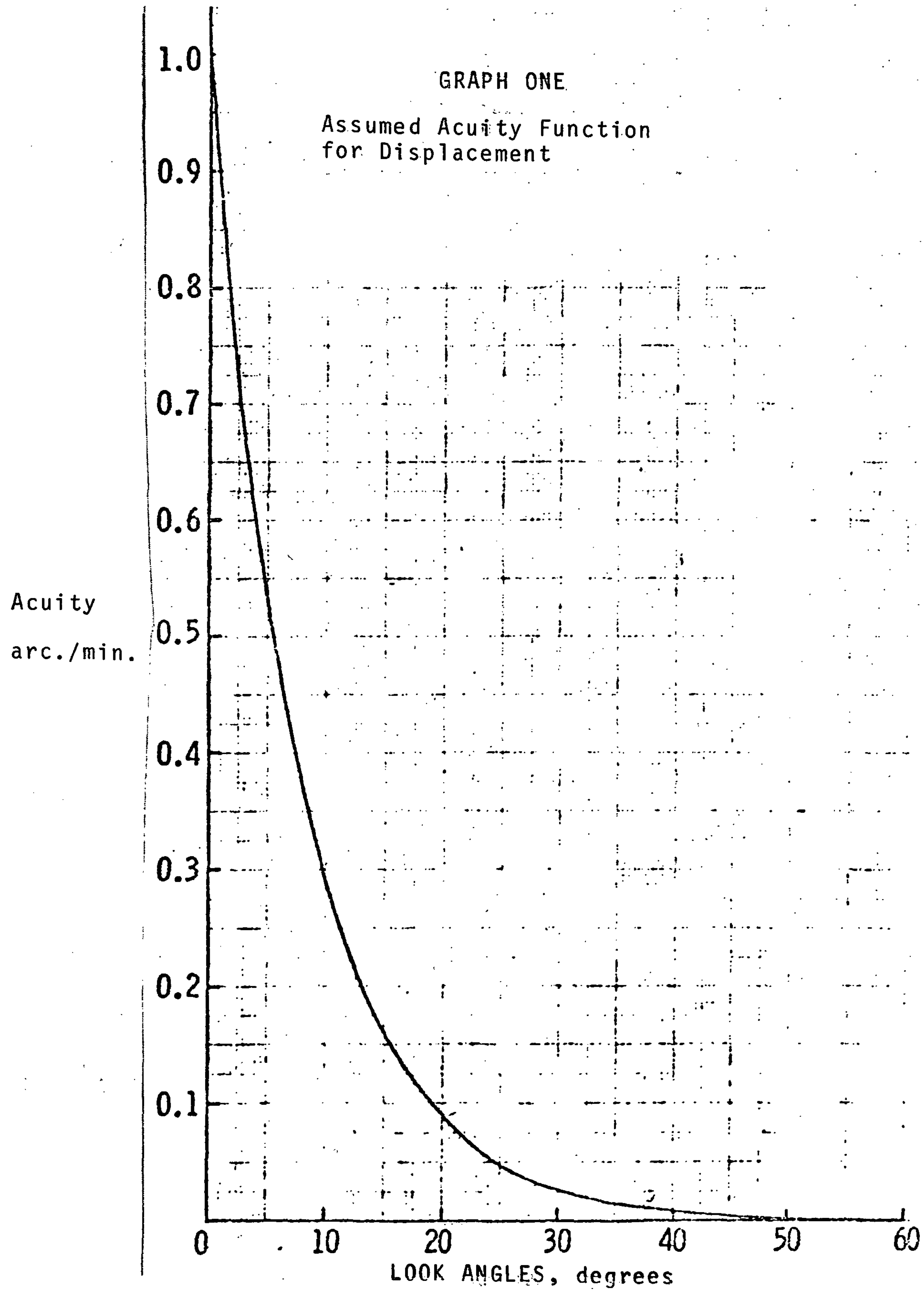
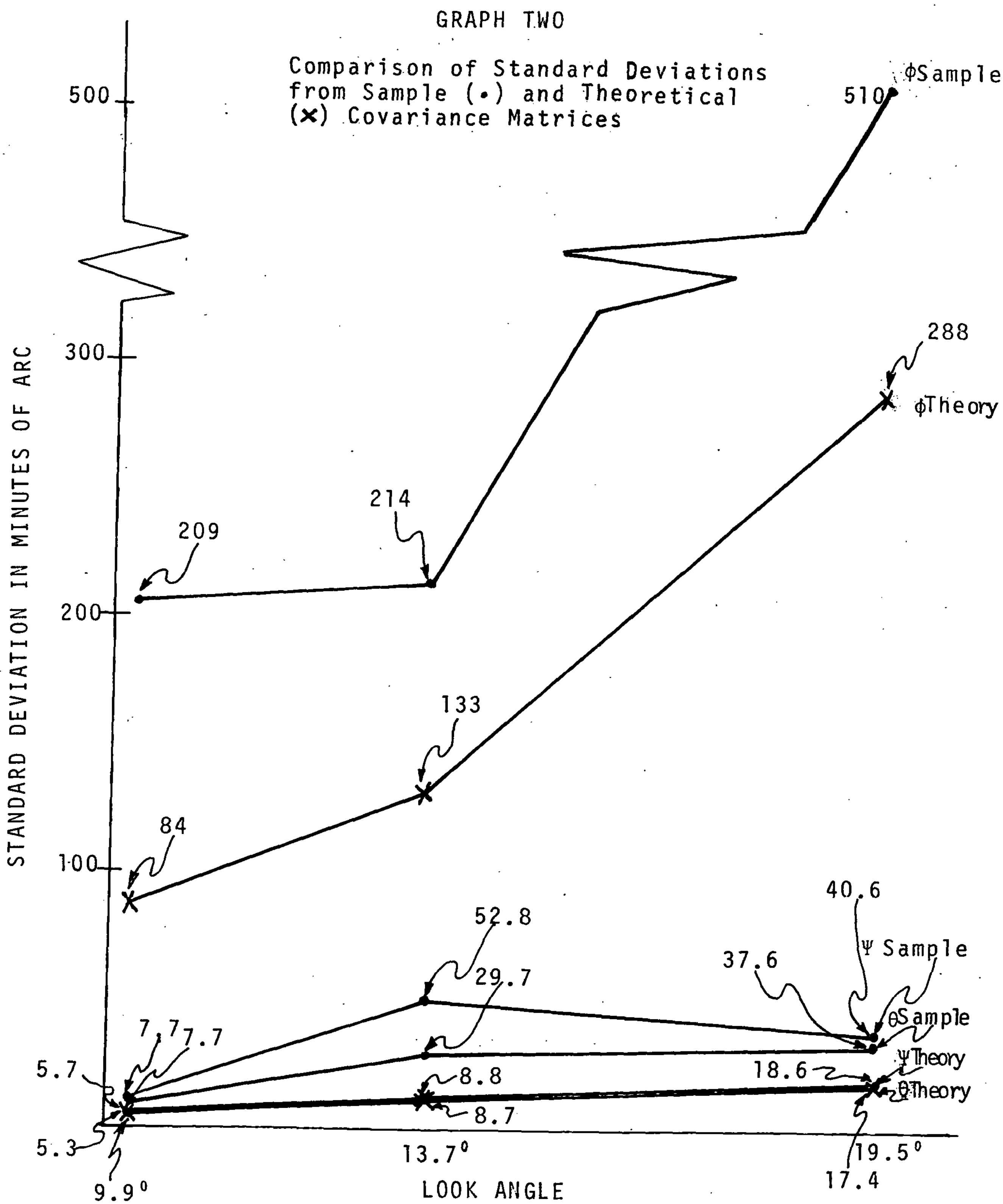


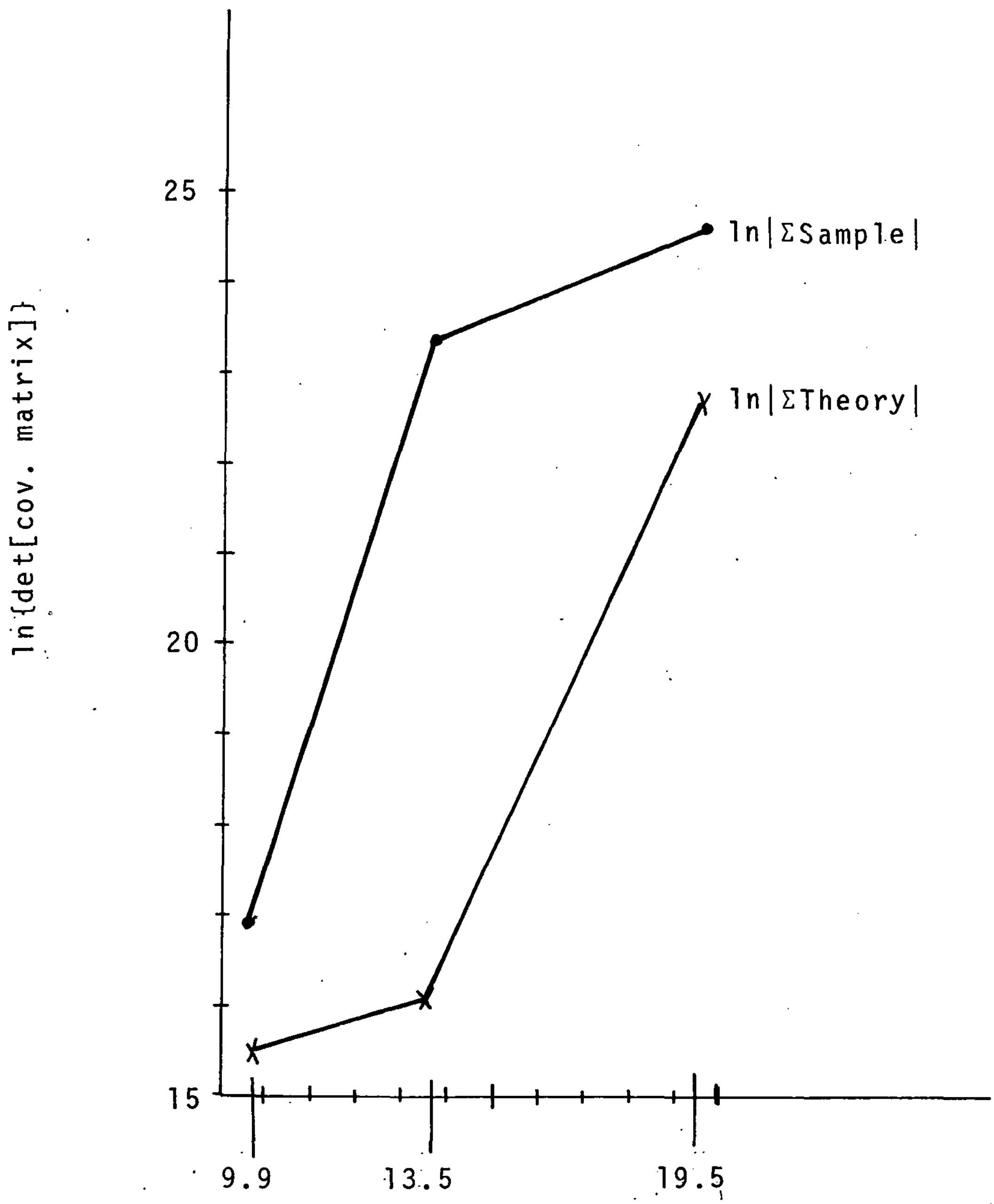
Figure 4B.- Fine Weights - Display B


GRAPH ONE
Assumed Acuity Function for Displacement
Acuity
arc./min.
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
LOOK ANGLES, degrees


GRAPH TWO
Comparison of Standard Deviations from Sample (•) and Theoretical (×) Covariance Matrices
STANDARD DEVIATION IN MINUTES OF ARC
500
300
200
100
510
φSample
288
φTheory
214
209
133
84
52.8
29.7
40.6
37.6
Ψ Sample
θSample
ΨTheory
θTheory
7.7
7.7
5.7
8.8
8.7
18.6
5.3
9.9°
13.7°
19.5°
17.4
LOOK ANGLE

GRAPH THREE

Comparison of Log Determinants of Sample and Theoretical Covariance Matrices



```
 TITLE CARD - EXAMPLE FOR USER≠S MANUAL - TWO DISPLAYS.
----------------------------------------------------------

NUMBER OF STATE VARIABLES                6

ESTIMATION IS FOR POSITION OF STATE VARIABLES

INPUT DATA IS IN DEGREES

WEIGHTS WILL BE READ IN

EIGENVECTORS WILL BE CALCULATED

OBSERVATION VECTOR WILL NOT BE PRINTED

INFLUENCE MATRICES WILL NOT BE PRINTED

STATE VECTOR CHANGES NOT CALCULATED.

   2 VISUAL SYSTEMS OR DISPLAYS ARE BEING CONSIDERED
   1TH VIEW COORDINATE SYSTEM OR DISPLAY IS   -10 DEGREES HORIZONTAL
 -20 DEGREES FROM VERTICAL.


***  FUNCTIONAL FORM     3 FOR THIS SYSTEM ***
           FORM 1 IS A REAL WORLD SPHERE,
           FORM 2 IS A CIRCULAR DISPLAY,
           FORM 3 IS A LINEAR DISPLAY.

   2TH VIEW COORDINATE SYSTEM OR DISPLAY IS    20 DEGREES HORIZONTAL
 -15 DEGREES FROM VERTICAL.


***  FUNCTIONAL FORM     1 FOR THIS SYSTEM ***
           FORM 1 IS A REAL WORLD SPHERE,
           FORM 2 IS A CIRCULAR DISPLAY,
           FORM 3 IS A LINEAR DISPLAY.
```

TABLE 1

OUTPUT SHOWING INITIAL DISPLAY SET-UP

```
*****  L O O K SYSTEM ***      1 *******************
LATIDUDINAL SEGMENT LENGTH               1.00000
LONGITUDINAL SEGMENT LENGTH              2.00000
MAXIMUM LATIDUDINAL ANGLE                1.00000
MAXIMUM LONGITUDINAL ANGLE               6.00000
LIMIT FOR CHANGES                        1.00000
AMOUNT OF CHANGE                         1.00000
EPSILON FOR DIFFERENTIATION                             .02000
DISPLAY PARAMETERS              -4.00000   500.00000         0


*****  L O O K SYSTEM ***      2 *******************
LATIDUDINAL SEGMENT LENGTH               5.00000
LONGITUDINAL SEGMENT LENGTH              5.00000
MAXIMUM LATIDUDINAL ANGLE               10.00000
MAXIMUM LONGITUDINAL ANGLE              15.00000
LIMIT FOR CHANGES                        1.00000
AMOUNT OF CHANGE                         1.00000
EPSILON FOR DIFFERENTIATION                             .02000
DISPLAY PARAMETERS                  0  -20.00000  1024.00000


****************  A C U I T Y   D E G R A D E D    DUE TO NON-REAL WORLD DISPLAY
FOR DISPLAY OR VIEW POINT NUMBER     2 DEGRADATION ID       .6827

LESS THAN ONE IS WORSE THAN REAL WORLD, GREATER THAN ONE IS IMPROVEMENT.


********************************************************
NUMBER OF ELEMENTS                      112
```

TABLE 2.- OUTPUT SHOWING DISPLAY PARAMETERS

```
LOOK SYSTEM        1
LATITUDINAL LOOK ANGLE LAMBDA BETA                  0
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA          -3.00000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)         1.00000

LOOK SYSTEM        2
LATITUDINAL LOOK ANGLE LAMBDA BETA                  0
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA                0
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          .61440

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM        1
LATITUDINAL LOOK ANGLE LAMBDA BETA            -10.00000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA          -23.00000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)         1.00000

LOOK SYSTEM        2
LATITUDINAL LOOK ANGLE LAMBDA BETA             20.00000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA          -15.00000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          .61440


WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY.      1
WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS
              1   2    3    4    5    6    7
         1    0 1.0    0    0    0    0    0
         2    0 1.0    0    0    0    0    0
         3    0 1.0    0    0    0    0    0
WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY.      2
WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS
              1   2    3    4    5    6    7
         4    0   0    0    0    0    0    0
         5    0  .5  1.0  1.0  1.0  .5    0
         6    0  .5  1.0  1.0  1.0  .5    0
         7    0  .5  1.0  1.0  1.0  .5    0
         8    0   0    0    0    0    0    0
```

TABLE 3

OUTPUT SHOWING THE LOOK POINTS AND SEGMENT WEIGHTS

INVERS OF COVARIANCE MATRIX

1.174555	-.000074	.000203	.401341	1.103295	-0
-.000074	1.033800	.372640	-.000217	-.000000	-1.099373
.000203	.372640	.144954	-.000000	.000217	-.399915
.401341	-.000217	-.000000	.539472	.373985	.000203
1.103295	-.000000	.000217	.373985	1.037488	-.000074
-0	-1.099373	-.399915	.000203	-.000074	1.170386

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

801.905439	.010607	.026997	-7.203086	-850.172693	-.033383
.010607	24451.594755	8860.455655	.031328	-.017817	25995.565592
.026997	8860.455655	3331.141602	.011006	-.053188	9461.081133
-7.203086	.031328	.011006	2.535905	6.745847	.033174
-850.172693	-.017817	-.053188	6.745847	902.630166	.021062
-.033383	25995.565592	9461.081133	.033174	.021062	27651.940992

TABLE 4.- INTERMEDIATE OUTPUT

EIGENVECTORS

.698386	-.000000	-.000118	-.204542	-.685871	-.000278
-.000300	.664614	-.341927	.000113	-.000011	-.664360
.000028	.241886	.939727	.000000	-.000035	-.241671
.286299	.000001	.000000	.958123	.005789	.000034
.655965	-.000000	-.000071	-.200407	.727700	-.000306
.000273	.706951	-.000082	-.000108	.000023	.707263

CHARACTERISTIC ROOTS

.420989	55327.875774	106.374174	2.662634	1703.987888	.427401

DETERMINANT OF THE COVARIANCE MATRIX = 4804679595.00000

CORRELATIONS AND STD. DEV. IN MINUTES OF ARC.

28.317935	.000002	.000017	-.159731	-.999288	-.000007
.000002	156.370057	.981761	.000126	-.000004	.999730
.000017	.981761	57.716043	.000120	-.000031	.985783
-.159731	.000126	.000120	1.592453	.140999	.000125
-.999288	-.000004	-.000031	.140999	30.043804	.000004
-.000007	.999730	.985783	.000125	.000004	166.288728

TABLE 5
FINAL OUTPUT FOR THE SPECIFIED LOOK POINTS

```
RUN WITH EXPERIMENTAL DATA GROSS WEIGHTS.
--------------------------------------------------------------------------

NUMBER OF STATE VARIABLES                    3

ESTIMATION IS FOR POSITION OF STATE VARIABLES

INPUT DATA IS IN DEGREES

WEIGHTS WILL BE READ IN

EIGENVECTORS WILL BE CALCULATED

OBSERVATION VECTOR WILL NOT BE PRINTED

INFLUENCE MATRICES WILL BE PRINTED

STATE VECTOR CHANGES NOT CALCULATED.

    1 VISUAL SYSTEMS OR DISPLAYS ARE BEING CONSIDERED .
    1TH VIEW COORDINATE SYSTEM OR DISPLAY IS     0 DEGREES HORIZONTAL
    0 DEGREES FROM VERTICAL.

***  FUNCTIONAL FORM     1 FOR THIS SYSTEM ***
          FORM 1 IS A REAL WORLD SPHERE,
          FORM 2 IS A CIRCULAR DISPLAY,
          FORM 3 IS A LINEAR DISPLAY.

*****  L O O K SYSTEM ***        1 ********************
LATIDUDINAL SEGMENT LENGTH                    1.00000
LONGITUDINAL SEGMENT LENGTH                   1.00000
MAXIMUM LATIDUDINAL ANGLE                     8.00000
MAXIMUM LONGITUDINAL ANGLE                    8.00000
LIMIT FOR CHANGES                             1.00000
AMOUNT OF CHANGE                              1.00000
EPSILON FOR DIFFERENTIATION                                     .02000
DISPLAY PARAMETERS                 0                0                0

NUMBER OF ELEMENTS                           289
```

TABLE 6.- SET-UP OF DISPLAY PARAMETERS

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 19.50000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 0
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 19.50000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 0
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000
WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY. 1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	.5	.5	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	1.0	0	0	0	0	0	0	0	0	1.0	0	0
6	0	0	0	0	0	0	1.0	0	0	0	0	0	.5	.5	0	0	0
7	0	0	0	0	0	0	.5	.5	0	0	0	1.0	0	0	0	0	0
8	0	0	0	0	0	0	0	1.0	0	.5	.5	-0	-0	-0	-0	-0	-0
9	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
10	0	0	0	0	0	0	.5	.5	0	1.0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	1.0	0	0	0	.5	.5	-0	-0	-0	-0	-0	-0
12	0	0	0	.5	.5	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0
13	0	0	1.0	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	0	0	0	.5	.5	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0
16	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 7

DISPLAY A - FIRST LOOK POINT PARAMETERS

INVERS OF COVARIANCE MATRIX

.201299	-.000244	.008481
-.000244	.200900	.001882
.008481	.001882	.001570

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

6.441619	.337508	-35.200277
.337508	5.051795	-7.877391
-35.200277	-7.877391	836.490029

EIGENVECTORS

.987523	.151688	-.042292
-.152213	.988302	-.009465
.040361	.015784	.999060

CHARACTERISTIC ROOTS

4.950916	4.977787	838.054740

DETERMINANT OF THE COVARIANCE MATRIX = 20653.5269847

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

2.538034	.059165	-.479533
.059165	2.247620	-.121180
-.479533	-.121180	28.922137

TABLE 8

DISPLAY A - FIRST LOOK POINT RESULTS

```
LOOK SYSTEM      1
LATITUDINAL LOOK ANGLE LAMBDA BETA            16.23000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA         -11.52000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)           1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM      1
LATITUDINAL LOOK ANGLE LAMBDA BETA            16.23000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA         -11.52000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)           1.00000
WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY.     1
```

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	.5	.5	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	1.0	0	0	0	0	0	0	0	0	1.0	0	0
6	0	0	0	0	0	0	1.0	0	0	0	0	0	.5	.5	0	0	0
7	0	0	0	0	0	0	.5	.5	0	0	0	1.0	0	0	0	0	0
8	0	0	0	0	0	0	0	1.0	0	.5	.5	-0	-0	-0	-0	-0	-0
9	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
10	0	0	0	0	0	0	.5	.5	0	1.0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	1.0	0	0	0	.5	.5	-0	-0	-0	-0	-0	-0
12	0	0	0	.5	.5	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0
13	0	0	1.0	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	0	0	0	.5	.5	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0
16	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 9

DISPLAY A - SECOND LOOK POINT PARAMETERS

INVERS OF COVARIANCE MATRIX

.206281	-.000479	.009343
-.000479	.205609	.006469
.009343	.006469	.001706

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

6.756920	1.339699	-42.082955
1.339699	5.787957	-29.281893
-42.082955	-29.281893	927.625346

EIGENVECTORS

.980291	.192240	-.045534
-.193760	.980537	-.031684
.038557	.039882	.998460

CHARACTERISTIC ROOTS

4.836909	4.859603	930.473711

DETERMINANT OF THE COVARIANCE MATRIX = 21871.2139435

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

2.599408	.214225	-.531552
.214225	2.405817	-.399623
-.531552	-.399623	30.456942

TABLE 10

DISPLAY A - SECOND LOOK POINT RESULTS

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA -10.46000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA -9.09000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA -10.46000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA -9.09000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY. 1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	0	0	0	0	0	0	0	1.0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	1.0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	1.0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	1.0	0	-0	-0	-0	-0	-0	-0	-0
6	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
7	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
8	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
9	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0
10	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
12	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
13	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
16	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 11

DISPLAY B - THIRD LOOK POINT PARAMETERS

INVERS OF COVARIANCE MATRIX

1.191854	-.000019	-.030008
-.000019	1.188379	.023546
-.030008	.023546	.007498

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

.940031	-.079470	4.011593
-.079470	.904031	-3.156897
4.011593	-3.156897	159.332139

EIGENVECTORS

.987187	.157551	.025298
-.157073	.987386	-.019908
-.028115	.015679	.999482

CHARACTERISTIC ROOTS

.838425	.841220	159.496555

DETERMINANT OF THE COVARIANCE MATRIX = 112.4929553

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

.969552	-.086206	.327789
-.086206	.950806	-.263037
.327789	-.263037	12.622683

TABLE 12

DISPLAY B - THIRD LOOK POINT RESULTS

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 8.96000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 4.22000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 8.96000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 4.22000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY. 1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
6	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
7	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
8	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
9	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0
10	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
12	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
13	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
16	0	0	0	0	0	0	0	0	1.0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 13

DISPLAY B - FOURTH LOOK POINT PARAMETERS

INVERS OF COVARIANCE MATRIX

3.122128	.000050	.112465
.000050	3.115620	-.032044
.112465	-.032044	.019845

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

.404205	-.023964	-2.329414
-.023964	.327805	.665126
-2.329414	.665126	64.666021

EIGENVECTORS

.993635	.106683	-.036176
-.106676	.994272	.010329
.037070	-.006416	.999292

CHARACTERISTIC ROOTS

.319665	.320942	64.757224

DETERMINANT OF THE COVARIANCE MATRIX = 6.6478464

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

.635771	-.065835	-.455625
-.065835	.572542	.144464
-.455625	.144464	8.041519

TABLE 14

DISPLAY B-- FOURTH LOOK POINT RESULTS

EXPERIMENTAL SET-UP DISTANCE TO DISPLAY = 165.10CM.
LOOK POINT LOCATION X = 0CM. Y = 59.44CM.
DIAGONAL CROSS STRAIGHT UP LOOK POINT, DISP. A LOOK PT. 1 SUBJ. 1

DATA POINT	OBSERVED DATA X (CM.)	Y (CM.)	A (CM.)	B (CM.)	ANGLES (MIN. OF ARC) PSI	THETA	PHI(=ATAN(A-B/10))
1	.8890	5.8420	.7620	.7620	18.5108	121.5926	0
2	0	2.5400	2.6670	2.5400	0	52.8842	109.1118
3	-3.5560	-1.3970	-1.5240	-.6350	-74.0323	-29.0879	-751.8195
4	3.0480	1.1430	.7620	1.5240	63.4589	23.7994	-647.1369
5	2.1590	-2.0320	-.2540	-1.9050	44.9526	-42.3086	1345.7032
6	-.2540	2.4130	3.5560	2.5400	-5.2888	50.2404	855.1045
7	.2540	3.8100	3.6830	4.0640	5.2888	79.3185	-326.4605
8	-.7620	.2540	.2540	.3810	-15.8664	5.2888	-109.1118
9	-1.2700	.5080	1.6510	.7620	-26.4437	10.5776	751.8195
10	-.1270	.6350	.5080	.4445	-2.6444	13.2220	54.5696
11	6.6040	-.3810	-.1270	-.5080	137.4366	-7.9332	326.4605

AVERAGE VALUES PSI = 13.2156 THETA = 25.2358 PHI = 13.2912

COVARIANCE MATRIX

PSI	THETA	PHI
2985.27839369	-99.10786825	7415.58307874
-99.10786825	2297.18809116	-6270.89025795
7415.58307874	-6270.89025795	09618.2843017*

TABLE 15

EXPERIMENTAL DATA AND RESULTS

DISPLAY A - LOOK POINT 1

CORRELATION MATRIX WITH STANDARD DEVIATION ON DIAGONAL.

PSI	THETA	PHI
54.63770121	-.03784578	.21206218
-.03784578	47.92899009	-.20442841
.21206218	-.20442841	640.01428445

EXPERIMENTAL SET-UP DISTANCE TO DISPLAY = 165.10CM.
LOOK POINT LOCATION X = 0CM. Y = 59.44CM.
DIAGONAL CROSS STRAIGHT UP LOOK POINT, DISP A, LOOK PT. 1 SUBJ. 1

	OBSERVED DATA				ANGLES (MIN. OF ARC)		
DATA POINT	X (CM.)	Y (CM.)	A (CM.)	B (CM.)	PSI	THETA	PHI(=ATAN(A-B/10))
1	5.0800	.2540	.5080	0	105.7435	5.2888	434.2690
2	-3.0480	-.8890	-2.0320	-2.9210	-63.4589	-18.5108	751.8195
3	1.6510	-.7620	-.6350	-.8890	34.3763	-15.8664	218.0042
4	2.5400	-1.1430	-.8890	-1.1430	52.8842	-23.7994	218.0042
5	-1.0160	.3810	.3810	-.2540	-21.1551	7.9332	541.2259
6	-.7620	-1.2700	-1.2700	-1.2700	-15.8664	-26.4437	0
7	-3.0480	.5080	-.1270	.5080	-63.4589	10.5776	-541.2259
8	-3.8100	.5080	0	.5080	-79.3185	10.5776	-434.2690
9	-.6350	0	.1270	-.1270	-13.2220	0	218.0042
10	0	1.0160	.5080	1.0160	0	21.1551	-434.2690
11	-.2540	-1.2700	-.2540	.8890	-5.2888	-26.4437	-956.8373
12	.1270	-1.0160	-1.0160	-1.1430	2.6444	-21.1551	109.1118

AVERAGE VALUES PSI = -5.5100 THETA = -6.3905 PHI = .8600

COVARIANCE MATRIX

PSI	THETA	PHI
2731.85356057	-151.56843638	7065.92057145
-151.56843638	298.09351911	-1063.05782455
7065.92057145	-1063.05782455	53235.8755722*

TABLE 16

EXPERIMENTAL DATA AND RESULTS

DISPLAY A - LOOK POINT 1

SECOND SUBJECT

CORRELATION MATRIX WITH STANDARD DEVIATION ON DIAGONAL.

PSI	THETA	PHI
52.26713653	-.16795920	.26864417
-.16795920	17.26538500	-.12235395
.26864417	-.12235395	503.22547190

EXPERIMENTAL SET-UP DISTANCE TO DISPLAY = 165.10CM.
LOOK POINT LOCATION X = -33.65CM. Y = 48.01CM.
DIAGONAL DISPLAY LOOK PT. LEFT UPPER, DISP A, LOOK PT. 2, SUBJ 1

	OBSERVED DATA				ANGLES (MIN. OF ARC)		
DATA POINT	X (CM.)	Y (CM.)	A (CM.)	B (CM.)	PSI	THETA	PHI(=ATAN(A=B/10)
1	0	-1.7780	-1.6510	-1.7780	0	-37.0205	109.1118
2	.1270	.8890	.7620	.8890	2.6444	18.5108	-109.1118
3	1.1430	1.6510	2.4130	1.6510	23.7994	34.3763	647.1369
4	0	-4.1910	-4.0640	-4.1910	0	-87.2471	109.1118
5	-.6350	.6350	1.0160	.6350	-13.2220	13.2220	326.4605
6	-.5080	-5.2070	-5.2070	-4.3180	-10.5776	-108.3853	-751.8195
7	0	-1.7780	-2.1590	-1.7780	0	-37.0205	-326.4605
8	.2540	.8890	1.1430	1.0160	5.2888	18.5108	109.1118
9	-1.1430	-2.5400	-2.5400	-3.0480	-23.7994	-52.8842	434.2690
10	1.1430	.5080	.6350	.7620	23.7994	10.5776	-109.1118
11	0	.7620	1.0160	.7620	0	15.8664	218.0042
12	-1.0160	1.5240	1.7780	1.1430	-21.1551	31.7321	541.2259

AVERAGE VALUES PSI = -1.1018 THETA = -14.9801 PHI = 8.3189

COVARIANCE MATRIX

PSI	THETA	PHI
223.07983596	234.32870218	-339.62530399
234.32870218	2328.79740208	11003.06274962
-339.62530399	11003.06274962	51468.4806900*

TABLE 17

EXPERIMENTAL DATA AND RESULTS

DISPLAY A - LOOK POINT 2

CORRELATION MATRIX WITH STANDARD DEVIATION ON DIAGONAL.

PSI	THETA	PHI
14.93585739	.32510936	-.05842635
.32510936	48.25761496	.58585023
-.05842635	.58585023	389.18951770

EXPERIMENTAL SET-UP DISTANCE TO DISPLAY = 165.10CM.
LOOK POINT LOCATION X = -26.42CM. Y = -30.48CM.
STRAIGHT CROSS HAIRS LOOK POINT LOWER LEFT, DISP B, LOOK PT 3, SUBJ 1

DATA POINT	OBSERVED DATA X (CM.)	Y (CM.)	A (CM.)	B (CM.)	ANGLES (MIN. OF ARC) PSI	THETA	PHI(=ATAN(A-B/10)
1	-6.3500	0	0	-.1270	-132.1559	0	109.1118
2	3.3020	1.7780	2.0320	1.3970	68.7458	37.0205	541.2259
3	0	0	.0635	0	0	0	54.5696
4	.5080	3.8100	3.8100	3.8100	10.5776	79.3185	0
5	-.3810	1.0160	1.0160	1.3970	-7.9332	21.1551	-326.4605
6	3.4290	1.0160	1.0160	1.0160	71.3891	21.1551	0
7	.0635	-.1270	-.1270	-.0635	1.3222	-2.6444	-54.5696
8	2.5400	.3810	.7620	.3810	52.8842	7.9332	326.4605
9	1.6510	3.1750	3.4925	3.4290	34.3763	66.1024	54.5696
10	.5080	-.0635	0	-.0635	10.5776	-1.3222	54.5696
11	.0635	2.1590	-2.0320	-2.1590	1.3222	44.9526	109.1118
12	-.3810	-.7620	-.7620	-.6350	-7.9332	-15.8664	-109.1118

AVERAGE VALUES PSI = 8.5977 THETA = 21.4837 PHI = 5.2741

COVARIANCE MATRIX

PSI	THETA	PHI
2793.82982624	488.14659403	3591.16362566
488.14659403	884.73723412	883.55013388
3591.16362566	883.55013388	45967.16165924

TABLE 18

EXPERIMENTAL DATA AND RESULTS

DISPLAY B - LOOK POINT 3

CORRELATION MATRIX WITH STANDARD DEVIATION ON DIAGONAL.

PSI	THETA	PHI
52.85669140	.31048680	.31689207
.31048680	29.74453284	.13854798
.31689207	.13854798	214.39953745

EXPERIMENTAL SET-UP DISTANCE TO DISPLAY = 165.10CM.
LOOK POINT LOCATION X = 12.19CM. Y = 26.04CM.
STRAIGHT CROSS LOOK POINT UPPER RIGHT, DISP B, POOK PT 4, SUBJ 1

	OBSERVED DATA				ANGLES (MIN. OF ARC)		
DATA POINT	X (CM.)	Y (CM.)	A (CM.)	B (CM.)	PSI	THETA	PHI(=ATAN(A-B/10)
1	-.7620	.5080	.6350	.5080	-15.8664	10.5776	109.1118
2	.1270	-.6350	-1.0160	-.6350	2.6444	-13.2220	-326.4605
3	-.2540	-.1270	0	-.1270	-5.2888	-2.6444	109.1118
4	-.0635	-.0635	0	-.0635	-1.3222	-1.3222	54.5696
5	-.7620	-.1270	0	-.1270	-15.8664	-2.6444	109.1118
6	.1270	.3810	.6350	.3810	2.6444	7.9332	218.0042
7	.5080	.2540	0	.3810	10.5776	5.2888	-326.4605
8	-.2540	-.1270	-.2540	-.1270	-5.2888	-2.6444	-109.1118
9	-.2540	.3810	0	.3810	-5.2888	7.9332	-326.4605
10	.0635	.2540	.3810	.2540	1.3222	5.2888	109.1118
11	0	.3810	.3810	.5080	0	7.9332	-109.1118
12	.1270	-.5080	-.2540	-.5080	2.6444	-10.5776	218.0042

AVERAGE VALUES PSI = -2.4240 THETA = .9917 PHI = -1.8790

COVARIANCE MATRIX

PSI	THETA	PHI
59.38654774	-8.02588956	-498.46376543
-8.02588956	59.47953725	17.88516915
-498.46376543	17.88516915	43917.27815247

TABLE 19

EXPERIMENTAL DATA AND RESULTS

DISPLAY B - LOOK POINT 4

CORRELATION MATRIX WITH STANDARD DEVIATION ON DIAGONAL.

PSI	THETA	PHI
7.70626678	-.13504091	-.30865395
-.13504091	7.71229779	.01106602
-.30865395	.01106602	209.56449640

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 19.50000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 0
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 19.50000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA 0
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY. 1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	.0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	.1	.1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	.1	0	0	0	0	0	0	0	.0	.0	0	0
6	0	0	0	0	0	0	.1	0	0	0	0	.1	.1	0	0	0	-0
7	0	0	0	0	0	0	.1	.1	0	0	.0	.1	.0	0	0	0	-0
8	0	0	0	0	0	0	0	.1	0	.1	.1	0	0	0	0	0	-0
9	0	0	0	0	0	0	0	.0	.3	.0	0	0	0	0	0	0	-0
10	0	0	0	0	0	0	.1	.1	.1	.1	0	0	0	0	0	0	-0
11	0	0	0	0	.0	.1	.1	0	0	.1	.0	0	0	0	0	0	-0
12	0	0	0	.1	.1	0	0	0	0	0	.1	0	0	0	0	0	-0
13	0	0	.0	.1	0	0	0	0	0	0	.0	.1	0	0	0	0	-0
14	0	0	0	0	0	0	0	0	0	0	0	.1	.0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	0	0	0	0	.0	0	0	0	-0
16	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 20
DISPLAY A - LOOK POINT 1 - FINE WEIGHTS

INVERS OF COVARIANCE MATRIX

.003363	-.000002	.000081
-.000002	.003357	.000028
.000081	.000028	.000014

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

346.043944	16.853035	-2013.294988
16.853035	303.591953	-689.030263
-2013.294988	-689.030263	83204.050762

EIGENVECTORS

.986409	.162507	-.024276
-.162748	.986633	-.008308
.022601	.012146	.999671

CHARACTERISTIC ROOTS

297.133567	297.885312	83258.667784

DETERMINANT OF THE COVARIANCE MATRIX = 7369363335.7500000

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

18.602256	.051996	-.375206
.051996	17.423890	-.137095
-.375206	-.137095	288.451124

TABLE 21.- RESULTS

DISPLAY A - LOOK POINT 1 - FINE WEIGHTS

LOOK SYSTEM 1

LATITUDINAL LOOK ANGLE LAMBDA BETA 16.23000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA -11.52000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM 1
LATITUDINAL LOOK ANGLE LAMBDA BETA 16.23000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA -11.52000
DEGRADATION FACTOR -(MEMORY AND DISPLAY) 1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY. 1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	.0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	.1	.1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	.1	0	0	0	0	0	0	0	.0	.0	0	0
6	0	0	0	0	0	0	.1	0	0	0	0	.1	.1	0	-0	-0	-0
7	0	0	0	0	0	0	.1	.1	0	0	.0	.1	.0	0	0	0	-0
8	0	0	0	0	0	0	0	.1	0	.1	.1	0	0	0	-0	-0	-0
9	0	0	0	0	0	0	0	.0	.3	.0	0	0	0	0	-0	-0	-0
10	0	0	0	0	0	0	.1	.1	.1	.1	0	0	-0	-0	-0	-0	-0
11	0	0	0	0	.0	.1	.1	0	0	.1	.0	0	0	0	0	0	-0
12	0	0	0	.1	.1	0	0	0	0	0	.1	0	0	0	0	-0	-0
13	0	0	.0	.1	0	0	0	0	0	0	.0	.1	0	0	0	-0	-0
14	0	0	0	0	0	0	0	0	0	0	0	.1	.0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	0	0	0	0	.0	0	-0	-0	-0
16	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 22

DISPLAY A - LOOK POINT 2 - FINE WEIGHTS

INVERS OF COVARIANCE MATRIX

.003324	-.000004	.000083
-.000004	.003318	.000071
.000083	.000071	.000016

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

353.654201	45.594824	-2105.464426
45.594824	340.201050	-1803.931407
-2105.464426	-1803.931407	83888.450872

EIGENVECTORS

.974778	.221753	-.025175
-.222301	.974739	-.021570
.019756	.026622	.999450

CHARACTERISTIC ROOTS

300.584788	301.305061	83980.416273

DETERMINANT OF THE COVARIANCE MATRIX = 7605914656.5000000

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

18.805696	.131449	-.386552
.131449	18.444540	-.337677
-.386552	-.337677	289.635030

TABLE 23

RESULTS - DISPLAY A - LOOK POINT 2 - FINE WEIGHTS

```
LOOK SYSTEM        1
LATITUDINAL LOOK ANGLE LAMBDA BETA          -10.46000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA         -9.09000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM        1
LATITUDINAL LOOK ANGLE LAMBDA BETA          -10.46000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA         -9.09000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY.      1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS
```

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	.1	0	0	0	0	0	0	0	0
9	0	.1	.1	.1	.1	.1	.1	.1	.2	.1	.1	.1	.1	.1	.1	.1	0
10	0	0	0	0	0	0	0	0	.1	0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
12	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
13	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
16	0	0	0	0	0	0	0	0	.1	0	-0	-0	-0	-0	-0	-0	-0
17	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 24

PARAMETERS - DISPLAY B - LOOK POINT 3 - FINE WEIGHTS

INVERS OF COVARIANCE MATRIX

.014019	-.000000	-.000267
-.000000	.013989	.000215
-.000267	.000215	.000065

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

77.753775	-5.162520	335.616581
-5.162520	75.649894	-270.608263
335.616581	-270.608263	17589.357355

EIGENVECTORS

.991468	.128939	.019153
-.128654	.991569	-.015443
-.020982	.012847	.999697

CHARACTERISTIC ROOTS

71.301085	71.472514	17599.967426

DETERMINANT OF THE COVARIANCE MATRIX = 89690626.6835936

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

8.816076	-.067321	.287021
-.067321	8.697695	-.234591
.287021	-.234591	132.624875

TABLE 25

RESULTS - DISPLAY B - LOOK POINT 3 - FINE WEIGHTS

```
LOOK SYSTEM      1
LATITUDINAL LOOK ANGLE LAMBDA BETA            8.96000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA          4.22000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          1.00000

ABOVE LOOK ANGLES ARE FOR VIEW COORDINATE SYSTEM, ORDISPLAY.
FOR AIRCRAFT COORDINATE SYSTEM THE FOLLOWING ARE THE LOOK ANGLES....

LOOK SYSTEM      1
LATITUDINAL LOOK ANGLE LAMBDA BETA            8.96000
LONGITUDINAL LOOK ANGLE LAMBDA ALPHA          4.22000
DEGRADATION FACTOR -(MEMORY AND DISPLAY)          1.00000

WEIGHTED CELLS FOR VISUAL SYSTEM OR DISPLAY.      1

WEIGHTS ARE READ-IN - WEIGHTED CELLS AS FOLLOWS
```

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	0	0	0	0	0	0	0	0	.1	0	-0	-0	-0	-0	-0	-0	-0
3	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
4	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
5	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
6	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
7	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
8	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
9	0	.1	.1	.1	.1	.1	.1	.1	.2	.1	.1	.1	.1	.1	.1	.1	0
10	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
11	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
12	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
13	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
14	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
15	0	0	0	0	0	0	0	0	.1	-0	-0	-0	-0	-0	-0	-0	-0
16	0	0	0	0	0	0	0	0	.1	0	-0	-0	-0	-0	-0	-0	-0
17	0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

TABLE 26

PARAMETERS - DISPLAY B - LOOK POINT 4 - FINE WEIGHTS

INVERS OF COVARIANCE MATRIX

.036449	.000000	.001002
.000000	.036389	-.000308
.001002	-.000308	.000173

E I G E N V E C T O R   A N A L Y S I S

COVARIANCE MATRIX

32.746483	-1.634370	-193.121449
-1.634370	27.983882	59.418238
-193.121449	59.418238	7022.484834

EIGENVECTORS

.995280	.093041	-.027597
-.092849	.995645	.008491
.028267	-.005889	.999583

CHARACTERISTIC ROOTS

27.414183	27.479729	7028.321292

DETERMINANT OF THE COVARIANCE MATRIX = 5294675.6857910

CORRELATIONS AND VARIANCES IN MINUTES OF ARC.

5.722455	-.053990	-.402720
-.053990	5.289979	.134036
-.402720	.134036	83.800268

TABLE 27

THEORETICAL RESULTS - DISPLAY B - LOOK POINT 4 - FINE WEIGHTS

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SUBJECT	DISPLAY [1]	LOOKPOINT [2]	WEIGHT MODE [3]	ψ σ_ψ^2	ψ S_ψ^2	ψ $(\chi^2)_4$	ψ $(P)_5$	θ σ_θ^2	θ S_θ^2	θ $(\chi^2)_4$	θ $(P)_5$
2	A	1	GROSS	6.442	2985.3	5099.0	.00001	5.052	2297.2	5004.0	.00001
2	A	1	FINE	346.0	2985.3	94.9	.0001	303.6	2297.2	83.6	.0001
1	A	1	GROSS	6.442	2731.9	4664	.00001	5.052	298.1	649.1	.00001
1	A	1	FINE	346.0	2731.9	86.9	.0001	303.6	298.1	10.8	.46
1	A	2	GROSS	6.757	223.08	363.	.0001	5.788	2328.8	4424.	.00001
1	A	2	FINE	353.7	223.08	6.94	.80	340.2	2328.8	75.3	.0001
1	B	3	GROSS	.9400	2793.8	32693.	.00001	.9040	884.7	10765.	.00001
1	B	3	FINE	77.7	2793.8	395.	.0001	75.6	884.7	128.7	.0001
1	B	4	GROSS	.4042	59.39	1616.5	.00001	.3278	59.48	1996.0	.00001
1	B	4	FINE	32.75	59.39	19.95	.047	27.98	59.48	23.4	.017

1,2,3,4,5, and 6. See following page.

TABLE

Comparison of Theoretical and Experimental

(Gross and

ϕ				Σ	
σ^2_ϕ	S^2_ϕ	$(\chi^2)_4$	$(P)_5$	$(\lvert\Sigma\rvert)_6$	$(\lvert\hat{\Sigma}\rvert)_6$
836.5	66805.5	878.5	.00001	2.065×10^4	4.188×10^{11}
832041	66805.5	8883	.64	7.369×10^9	4.188×10^{11}
836.5	40250.8	529.3	.00001	2.065×10^4	2.943×10^{10}
832041	40250.8	5.32	.91	7.369×10^9	2.943×10^{10}
927.6	23909.8	283.5	.0001	2.187×10^4	9.12×10^9
[illegible]	23909.8	3.14	.988	7.605×10^9	9.12×10^9
159.33	7192.0	496.5	.00001	1.125×10^2	1.44×10^{10}
[illegible]	7192.0	4.5	.952	8.969×10^6	1.44×10^{10}
64.67	6832.6	1162.2	.00001	6.648	2.141×10^7
[illegible]	6832.6	10.7	.471	5.295×10^6	2.141×10^7

28
Results for Each Display and Look Point
Fine Weights)

1. Display A is a crosshairs at 35° from horizontal; display B is the horizontal crosshair. Both are 14° x 14° with line width of .11 degrees (See Figures 2A, 2C).
2. Lookpoint 1 = (0°, 19°); lookpoint 2 = (-11.52°, 16.23°); 3 = (-9.09°, -10.46°); and 4 = (4.22°, 8.96°) (See Figures 2A, 2C).
3. Gross weights are 0, .15, or 1 depending on relative line length in a segment (See Figures 2C, 2D). See Figures 3A and 3B for fine weights.
4. χ^2 test for sample variance = theoretical variance, $^2 = \frac{(N-1)s^2}{2}$ with (N-1) degrees of freedom.
5. P values from χ^2 table with 11 degrees of freedom, p = probability of as large or larger χ^2 value.
6. $|\Sigma|$ = determinant of theoretical covariance matrix, $\hat{\Sigma}$ is the sample covariance matrix.

(Continued from TABLE 28)

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TABLE

Comparison of Sample and Theoretical

SUBJECT	DISPLAY[1]	LOOKPOINT[1]	WEIGHT	MODE[1]	$\rho_{\psi\theta}$	$r_{\psi\theta}$	z^2	P_z
2	A	1	GROSS		.059	-.038	.336	.37
2	A	1	FINE		.052	-.038	.311	.38
1	A	1	GROSS		.059	-.168	.792	.21
1	A	1	FINE		.052	-.168	.768	.22
1	A	2	GROSS		.214	.325	-.416	.66
1	A	2	FINE		.131	.325	-.711	.76
1	B	3	GROSS		-.086	.310	-1.41	.92
1	B	3	FINE		-.067	.310	-1.35	.91
1	B	4	GROSS		-.066	-.135	.242	.41
1	B	4	FINE		-.054	-.135	.283	.39

1. See Table 28 for the entries in this column.
2. $1/2 \ln(\frac{1 + r}{1 - r})$ has an approximate
   In this case $\sigma_r^2 = 1/12$, so that z

3. P_z is the probability of a z-value

29

Correlations for Different Weighting Modes

$\rho_{\psi\theta}$	$r_{\psi\theta}$	z^2	P_z	$\rho_{\theta\phi}$	$r_{\theta\phi}$	z^2	P_z
-.480	.212	-2.56	.995	-.121	-.210	.317	.37
-.375	.212	-2.11	.983	-.137	-.210	.261	.40
-.480	.265	-2.75	.997	-.121	-.119	-.0095	.504
-.375	.265	-2.31	.990	-.137	-.119	-.066	.53
-.532	-.056	-1.86	.969	-.400	.587	-3.8	.9999
-.387	-.056	-1.22	.889	-.338	.587	-3.55	.9998
.328	.318	.039	.48	-.263	.139	-1.42	.922
.287	.318	-.117	.55	-.235	.139	-1.31	.905
-.456	-.309	-.598	.73	.144	.011	.466	.32
-.403	-.309	-.373	.64	.134	.011	.429	.33

normal distribution $N(\mu_r, \sigma_r)$ where $\mu_r = 1/2 \ln(\frac{1+\rho}{1-\rho})$, $\sigma_r^2 = 1/n$.

$$= \frac{1/2[\ln(\frac{1+\rho}{1-\rho}) - \ln(\frac{1+r}{1-r})]}{1/\sqrt{12}}$$ has the standard normal distribution,

this large or larger.